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# The Effect of Swing Leg Retraction on Biped Walking Stability Is Influenced by the Walking Speed and Step-length

Ruizhi Bao

Tao Geng

**Abstract** — Swing Leg Retraction (SLR) is observed in human walking and running. Previous studies have concluded that SLR improves the stability and robustness of biped walking. But this conclusion was based on analysis of robot models that can only walk at a very small range of step-lengths and slow or fixed speeds. By contrast, humans can walk with a large range of speeds and step-lengths. Moreover, human walking patterns have a special feature that has not been considered in the previous studies on SLR effects: At a given walking speed,  $v$ , humans prefer a step-length,  $s$ , which satisfies the power law,  $s \sim v^\beta$ . Therefore, previous studies on SLR can't tell us whether their conclusion will still hold in the full range of human walking patterns (i.e., various walking speeds and step-lengths). This is the question we want to answer in this paper. In this study, using a simple biped model, we studied how the SLR affects the walking stability in the full range of human walking speeds/step-lengths. Preliminary analysis of both models suggests the same conclusion: (1) SLR improves the stability more evidently in human-preferred walking patterns than in other walking patterns. (2) In walking patterns that are very unlike human-preferred ones, the SLR improves the stability very little, or even deteriorates it drastically. Therefore, the new finding of our study is that how the SLR affects the biped walking stability depends on the walking speed and step-length. SLR does not always improve the stability of biped walking.

**Index Terms**— Biped robots, Swing leg retraction, Human walking.

## I. INTRODUCTION

In human walking and running, the swing leg rotates forward first and then, at the end of the swing phase, is braked and rotates backward prior to heel-strike. This so-called Swing Leg Retraction (SLR) is also observed in animal gaits. The benefits of swing leg retraction include: increasing the viability and controllability regions, reducing energetic cost, impact force, and the risk of slippage at heel-strike, etc. [1]. In addition to these gains, previous studies have also shown that SLR improves the stability and robustness of biped walking. Hobbelen and Wisse studied how SLR affects the stability of biped walking using three models: a point mass simulated model, a realistic simulated model, and a physical prototype. Each of these models walked at a fixed step-length and a fixed

speed. Using the eigenvalues of the Poincare map as the measure of the stability in these models, they found that mild SLR velocities improve the stability of biped walking [2]. Other studies on this issue have used similar methods and obtained similar conclusions [1][3][4]. For application, some biped robots have employed SLR in their motion planning and control [12].

However, these studies have not adequately considered other determinant features of biped walking gait, such as the walking speed and step-length (another important parameter, the step-frequency, is determined by the step-length and walking speed). As has been demonstrated in many studies in human and biped walking (e.g., [11]), the walking speed and step-length determine the energetic and stability properties of walking gaits. While the simulated models and physical models used in the above-mentioned studies can only walk at a fixed or very small range of walking speed and medium step-length, humans can walk at a large range of walking speeds and step-lengths. In human walking, the typical ranges of walking speeds and step-lengths are 0.7 – 1.9 m/s and 0.4 – 0.7 m, respectively [5]. The previous studies on SLR can't tell us whether their conclusion will still hold in other walking patterns (e.g., human's fast walking with large step-length, which was not achievable in their models). Therefore, the research question we ask in this study is: How does the SLR affect the stability of biped walking patterns in the full range of human walking speed and step-length? In other words, does SLR improve the stability of biped walking at any combination of walking speed and step-length?

On the other hand, there is a special relationship between the walking speed and step-length in human walking, which has not been embodied in the biped models or robots used in the above-mentioned SLR studies. Theoretically, numerous walking patterns can be obtained by arbitrarily selecting the walking speed and step-length from the above-mentioned typical ranges. But this is not the case of human walking. In human normal walking gait, the step-length and walking speed are not independent to each other. At a given walking speed, humans have a preferred step-length [6]. It has been empirically found that the step-length,  $s$ , and the walking speed,  $v$ , obey the power law,  $s \sim v^\beta$ , in human walking. The value of  $\beta$  is around 0.42 for adults [6]. This relationship is commonly posited as a basic feature of human walking gait. In the literature, there are two hypotheses explaining this phenomenon. One is that human selects the preferred step-length to reduce the metabolic cost of walking [7]. The other hypothesis is that the preferred step-length optimizes the stability of head and pelvic acceleration [8]. As will be

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described below, our study in this paper might hint a third explanation for this.

The central issue we must consider first in this study is what kind of model we should use in the analysis of the SLR effects. Although real robots are more convincing for studying biped walking, they are not appropriate for this study, because none of today's real biped robots can demonstrate the full range of human walking speeds and step-lengths. Instead, in the studies of biped walking, simple models are more convenient for parametric analysis and can still disclose the underlying principles if they embed features that are indispensable to the research question of interest. As mentioned above, in previous studies of SLR in biped walking, there were two types of models. One is the very simple single mass-point model where the point mass represents the torso and the two legs are mass-less [4]. Because about 37% of the human body mass are on the legs, the closely coupled mechanics of the two legs in human walking could have dominant effect on the dynamics of walking, especially when fast SLR happens. This effect can't be represented in the mass-less leg model. The other type of model is realistic robot models as used in [2], which has full degree of freedoms of biped robots. But their mass distribution is not very similar to human's. More importantly, due to the lack of a powerful ankle push-off, all these simple models can't demonstrate the full ranges of walking speed and step-length of humans. More complex robots equipped with more degrees of freedom and sophisticated controllers are capable of walking with a larger variety of walking patterns. But, because their controllers play a dominant role in the robot walking, they might overwhelm the effect of SLR, making it difficult to identify how the SLR has contributed to the stability of the walking pattern. Moreover, the high dimensionality of the robot and the complex controller cause difficulty in analysis. Therefore, an ideal model for this study should be capable of generating the full range of human walking patterns (i.e., walking speeds and step-lengths) by varying a small number of model parameters. By modifying and combining some simple models available in literature for biped dynamics and control, we have obtained a simulated model that satisfies this ideal condition.

In this study, we simulated a simple biped model, which have mass distributions similar to human's and can reach the full range of speeds and step-lengths of human walking. Using this model, we analysed the effects of SLR on the stability of typical walking patterns covering the full range of human's walking speeds and step-lengths. The results imply that SLR improves the stability of biped walking patterns more evidently in walking patterns that are similar to human's preferred combinations of walking speeds and step-lengths. In walking patterns that are very unlike human-preferred ones (e.g., walking at fast speed while keeping a very small step-length, or walking at small speed while keeping a large step-length), the SLR improves the stability very little, or even deteriorate it drastically.

The paper is organized as follows. The configuration of the simulated model is briefly described in section II. The section III presents the dynamics equations and simulation results of the model. Section IV concludes the paper.

## II. THE MODEL

The model is a two-link model similar to the well-studied compass biped model, but with two links rather than just two mass points at the legs (see Fig. 1(A)). The mass distribution of the biped is similar to that of human body. One point mass at the hip represents the torso. Two links represent the legs that have rotational inertia and 37% of the total mass (the same percentage of mass in human legs). Similar to a typical human adult, the total mass of the model is 80 kg, and the leg-length is 0.9 m. In order to generate various walking speeds with minimal control, the biped model is put on a slope. By changing the inclination angle of the slope ( $\alpha$  in Fig. 1(A)), the biped walking speed can be conveniently changed in a large range. To control the step-length and SLR velocity, a controller is used at the swing leg (angle  $\theta$  in Fig. 1(A)) to track a pre-planned trajectory. Details of the dynamics equations and control will be described in the next section.

In selecting the model, our emphasis here is on the simplicity and convenience of analysis, rather than on the physical realizability (e.g., foot clearance is not realized in these models). Similar simple models have been widely used in studies that intended to uncover the basic principles of the dynamics in human walking and bipedal robots.

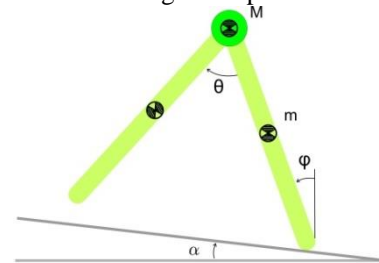


Fig. 1 The two-link compass biped model.

## III. THE DYNAMICS AND SIMULATION ANALYSIS OF THE TWO-LINK MODEL

In the simulation, each walking step of the biped model starts immediately when the stance leg leaves ground and becomes the swing leg, and terminates when the swing leg strikes on the ground. Therefore, a walking step involves two stages:

- (1) Stance phase: one foot is on the floor while the other foot is swinging in the air. The system is in a continuous state during this phase.
- (2) Landing stage: when the swing heel lands, it has impact with the floor. This is a discrete transient phase.

Below, we describe the dynamics equations of these two phases in subsection III.A and III.B, and then combine all these equations to get the computational model of a whole gait cycle in subsection III.C. Subsection III.C and III.D also describe how to obtain and analyze the nine typical walking patterns covering a large range of walking speeds and

step-lengths.

### A. The stance phase

With the Lagrange method, the equations that govern the motion of the simulated biped model in its stance phase are described as:

$$D(q)\ddot{q} + C(q, \dot{q}) + G(q) = \tau \quad (1)$$

where  $q = [\varphi(t), \theta(t)]^T$  is a vector describing the configuration of the biped (for the definition of  $\varphi$  and  $\theta$ , please see Fig. 1(A)),  $D(q)$  is a  $2 \times 2$  inertia matrix,  $C(q, \dot{q})$  is a  $2 \times 1$  vector of centripetal and coriolis forces,  $G(q)$  is a  $2 \times 1$  vector representing gravity forces,  $\tau = [\tau_0, \tau_1]^T$ ,  $\tau_0, \tau_1$  are the torques applied on the stance foot and the stance hip, respectively (see  $\varphi$  and  $\theta$  in Fig. 1(A)). Because there is no actuator at the foot/ankle, the torque around the ground contact point of the stance foot is zero ( $\tau_0 = 0$ ).  $\tau_1$  will be determined with a controller (i.e., a variant of computed torque control [10]) to drive the swing leg ( $\theta$  in Fig. 1(A)) to track a planned desired trajectory, moving forward first and then retracting before heel-strike. Therefore, the key issue in the stance phase is how to plan the swing leg trajectory.

Because the swing leg has substantial mass, we can't arbitrarily plan its trajectory to get SLR, as one of the previous SLR studies has done with mass-less legs [4]. In order to get the forward swing and retraction of the swing leg during the stance phase, we use 2-knot spline functions to construct the desired trajectories of the actuated joint ( $\theta$ ), which is similar to previous SLR studies [3] and many other motion planning studies on biped walking robots,

$$\theta_d(t) = \begin{cases} a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0 & 0 \leq t < 0.8T \\ b_3(t-0.8T)^3 + b_2(t-0.8T)^2 + b_1(t-0.8T) + b_0 & 0.8T \leq t < T \end{cases} \quad (2)$$

Where  $T$  is the duration of the walking step. The motion of the swing leg during the stance phase involves two sub-phases: (1) swinging forward sub-phase, from  $t=0$  to  $t=0.8T$  (the first equation in equation (2)); (2) SLR sub-phase, from  $t=0.8T$  to  $t=T$  (the second equation in equation (2)). The duration of the SLR sub-phase ( $0.2T$ ) is similar to that in previous SLR studies and human walking where the SLR sub-phase takes about 20% of the duration of the stance phase [2][4][5]. This spline function can be uniquely defined by specifying the values of  $T$ , positions and velocities at initial time ( $t=0$ ), knots ( $t=0.4T$ ,  $t=0.8T$ ), and final time ( $t=T$ ). All these values are known or can be calculated with the specified step-length, walking speed, SLR velocity (e.g.,  $T = \text{step-length}/\text{speed}$ ). The SLR velocity is the final value of  $\dot{\theta}$  at time  $t=T$ . Constraints such as positive vertical ground reaction force and non-slipping at the stance foot are also involved in the model of the stance phase.

Because the stance leg (angle  $\varphi$  in Fig. 1(A)) moves passively during this phase, even if the swing leg has been controlled to track the desired trajectory perfectly in equation (2), there will be three possible final states at time  $t=T$  in the simulation: (1) The swing foot touches ground exactly; (2) The swing foot is above the ground; (3) The swing foot has "penetrated" in

ground. Obviously, only the first possible state leads a cyclic walking gait. Which possible state will come true at time  $t=T$  is determined by the slope angle and the initial state of the biped at time  $t=0$ . Subsection III.C will present the method of searching for the proper initial condition and slope angle that lead to the desired first possible state.

### B. Swing leg landing phase

At time  $t=T$ , the swinging foot touches the floor and the transient landing phase starts. During this stage, the configuration of the robot,  $q$ , doesn't change. The strike of the swing heel is assumed to be an inelastic impact. This assumption implies the conservation of angular momentum of the robot just before and after the strike, with which the value of  $\dot{q}$  just after the strikes (i.e.,  $\dot{q}^+$ ) can be computed using the system state just before the strikes,  $\dot{q}^-$ . So, we have,

$$\dot{q}^+ = Q(q^-, \dot{q}^-) \quad (3)$$

Just after the landing of the swing foot, the two legs swap their roles. The initial state of the system at the beginning of the next walking step (i.e., just after the previous swing leg lands),  $\bar{x}_0 = [\bar{\varphi}_0, \bar{\theta}_0, \bar{\dot{\varphi}}_0, \bar{\dot{\theta}}_0]^T$  can be obtained as

$$\bar{x}_0 = E \begin{bmatrix} q^+ \\ \dot{q}^+ \end{bmatrix} \quad (4)$$

Where  $E$  is a matrix representing the role-swapping of the two legs. Once the swing leg lands, both legs are on the ground, supporting the body. Following many other studies on the compass biped model, the double support stage is assumed to be instantaneous and takes no time, and thus is ignored here. After the swing leg landing phase, the stance phase of the next walking step starts immediately.

### C. Searching and analyzing the walking patterns

By combining all the computations of equations (1) – (4), we can get the computational relationship between the initial state of the current walking step,  $x_0 = [\varphi_0, \theta_0, \dot{\varphi}_0, \dot{\theta}_0]^T$ , and the initial state of the next walking step,  $\bar{x}_0 = [\bar{\varphi}_0, \bar{\theta}_0, \bar{\dot{\varphi}}_0, \bar{\dot{\theta}}_0]^T$ :

$$\bar{x}_0 = f(x_0) \quad (5)$$

Obviously, a cyclic walking pattern (gait) is defined by a fixed point (root) of the following equation:

$$x^* = f(x^*) \quad (6)$$

Unknown variables are the fixed point,  $x^*$ , and the inclination angle of the slope,  $\alpha$ . Using the first order Newton shooting method described in [11], we can find these variables, and thus get the walking pattern that has the walking speed, step-length, and SLR velocity that were specified in subsection III.A. For the stability measure, we use the eigenvalues of the Poincare map of the fixed point (i.e., walking pattern) [11] [2][4].

**To summarize**, the procedures of the simulation analysis are:

- (1) Specify the three parameters that define the walking pattern: (a) walking speed; (b) step-length; (c) the SLR velocity at the end of the stance phase.
- (2) Use these parameters to obtain the desired trajectory of the swing leg in the stance phase (see equation (2)).

- (3) Combine the equations in subsection III.A and III.B, obtaining the computational model in equation (6).
- (4) Search the fixed point of the Poincare map, equation (9).
- (5) Calculate the eigenvalues of the Poincare map.

#### D. Results

The ranges of walking speed and step-length achievable in this two-link model are 0.6 – 1.6 m/s and 0.3 – 0.7 m, respectively. These are similar to the above-mentioned typical ranges in human walking, although the upper limit of typical human walking speed, 1.9 m/s, is beyond this range. We selected the following three walking speeds from this range:

**Slow speed, 0.6 m/s**

**Medium speed, 1.1 m/s**

**Fast speed, 1.6 m/s**

With each of these walking speeds, we get its corresponding step-length preferred by humans using the power law ( $s \sim v^\beta$ ), which are:

**Small step-length (preferred at slow speed, 0.6 m/s), 0.40 m**

**Medium step-length (preferred at medium speed, 1.1 m/s), 0.52 m**

**Large step-length (preferred at fast speed, 1.6 m/s), 0.61 m**

By combining these walking speeds and step-lengths, we get nine typical walking patterns covering the full achievable ranges of walking speeds and step-lengths. Each of these walking patterns is labelled with a number, as shown in Table 1. The walking patterns (1), (5), and (9) are corresponding to slow speed with small step-length, medium speed with medium step-length, and fast speed with large step-length, respectively. These three patterns are human-preferred walking patterns (see Table 1).

**Table 1. The walking speeds and step-lengths of the nine walking patterns chosen for analysis. The green shadowed ones are those preferred in human walking.**

	Slow speed (0.6 m/s)	Medium speed (1.1 m/s)	Fast speed (1.6 m/s)
Small step-length (0.40 m)	<b>(1)</b>	<b>(2)</b>	<b>(3)</b>
Medium step-length (0.52 m)	<b>(4)</b>	<b>(5)</b>	<b>(6)</b>
Large step-length (0.61 m)	<b>(7)</b>	<b>(8)</b>	<b>(9)</b>

With each of these nine walking patterns and SLR velocities in the range of 0 – 3 rad/s, we run the gait searching and stability analysis procedure described in subsection III.C. The magnitudes of the four eigenvalues of the nine walking patterns with various SLR velocities are shown in Fig. 3. In each walking pattern, there is a dominant eigenvalue (the red lines in Fig. 3). We use this eigenvalue as the stability criterion. The nine plots in Fig. 3 clearly show that the effect of SLR on the gait stability is closely related to the walking speed and step-length. The effects of the SLR on the stability can be classified with three categories (see Fig. 3):

- (1) In all the three human-preferred walking patterns at slow, medium, and fast walking speeds (see plot (1), (5) and (9) in Fig. 3), the dominant eigenvalues decrease evidently when the SLR velocity increase from zero, and reaches their minimum values when the

SLR velocities are in the range of 1.0 – 1.5 (mild value). This is consistent with the results of previous studies [2][4], although those studies have not involved such diverse walking patterns in terms of walking speeds and step-lengths.

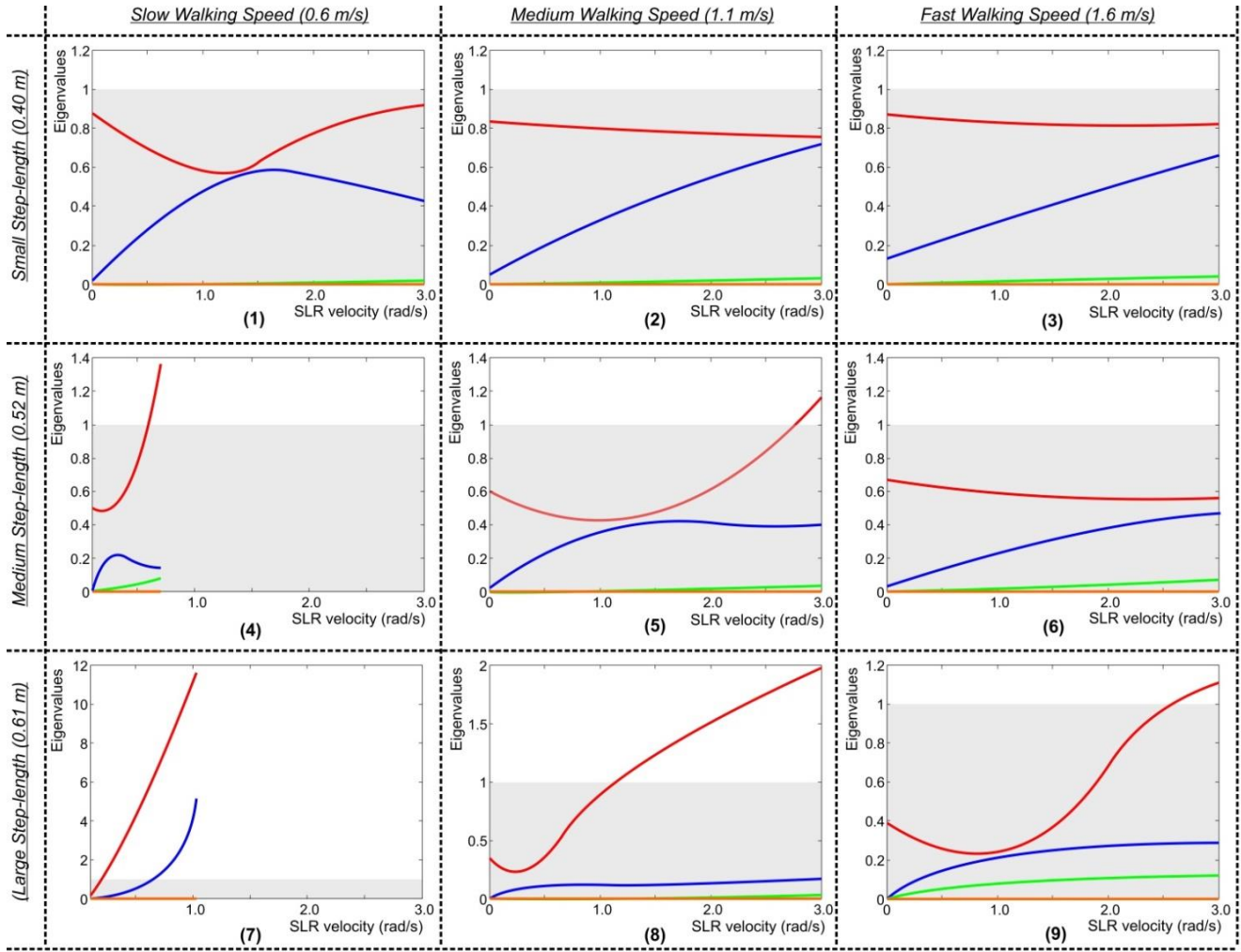
- (2) The SLR affects the dominant eigenvalues very little in the following three walking patterns in the upper right area in Fig. 3: (a) small step-length with medium speed (plot (2)); (b) small step-length with fast speed (plot (3)); (c) medium step-length and with fast speed (plot (6)).
- (3) The SLR deteriorate the stability in walking patterns with small step-length and medium (plot (4)) or fast (plot (7) in Fig. 3) speed.

As the Poincare map method is based on the linearization in an area around the fixed point in the state space, it is not valid to estimate the capability of the system to resist large disturbances. Another method for testing the robustness (disturbance resistance) of biped walking is to let it walk down a step and see how quickly it will recover. We applied this test on the three human-preferred walking patterns (number (1), (5), and (9) in Table 1; plot (1), (5), and (9) in Fig. 3). For each of these three walking patterns, we tested two cases: no SLR (SLR velocity is zero) and mild SLR velocities that lead to minimum magnitudes of eigenvalues in Fig. 3 (plot (1), (5), and (9)). Therefore we get six phase plots with these tests for the absolute angle and velocity of one leg (see Fig. 4). The down-step height for each of these three walking patterns was chosen to be the maximum one from which the biped can recover in both cases (i.e., with and without SLR). As shown in Fig. 4(A) and (B) (the two phase plots of the slow-speed and small-step-length walking pattern with and without SLR), after walking down a step of the same height, the walking pattern without SLR (Fig. 4(A)) takes more cycles to converge (recover) than the walking pattern with mild SLR does (Fig. 4(B)), although they have the same walking speed and step-length (see number 1 in Table 1). The same observation can be seen in the walking patterns of medium speed (Fig. 4(C) and (D)) and fast speed (Fig. 4(E) and (F)). These test results indicate that a mild SLR velocity improves the robustness of human-preferred walking patterns at all three typical walking speeds (slow, medium, and fast). This is consistent with and complementary to the conclusion of previous studies using different robot models walking only at fixed speeds [2].

#### IV. CONCLUSION AND DISCUSSION

Previous studies on the SLR in biped walking have evidenced its improvement of the stability by analyzing biped models walking at fixed speeds and step-lengths. We started this study by asking a more in-depth question: does this conclusion hold in the much larger range of human walking speeds and step-lengths? After analyzing 9 different walking patterns of a biped model covering the full range of human walking patterns (i.e., speeds and step-lengths), our answer to the question is, no. The SLR improves the stability more evidently in walking patterns similar to human-preferred





**Fig. 3** The magnitude of eigenvalues of the poincare maps in the nine walking patterns while the SLR velocity is increased from 0 to 3 rad/s. The shadowed areas are the stable regions where the maximum magnitude of eigenvalues are below 1. The curves are shorter in plot (4) and (7), because cyclic gait can't be found at higher SLR velocities.

ones, and affects the stability very slightly or even deteriorates it in walking patterns that are very different from human-preferred ones.

The conclusion of this study could be useful in the following two research areas:

#### (1) Biomechanics of human walking

Human's preference of a specific step-length at a given walking speed has been regarded as a basic feature of human walking. In the biomechanics literature, there are already two explanations for this phenomenon (i.e., reducing metabolic cost and stabilizing the head motion). Our results hint a third explanation: the walking stability is more likely to be improved by SLR at the human-preferred walking patterns than at other non-preferred walking patterns.

#### (2) Bipedal walking robots

Although our analysis of the models was mainly considering the range of human walking speeds and step-lengths, our new findings about the relevance between the SLR effects and the walking speed/step-length is obviously also useful to bipedal robotics, because the versatile human walking gait is the design target of many biped robots. Due to the nature of the

muscular system, it would cause fatigue and cost extra energy if humans kept the swing leg at a fixed angle at the end of the swing phase without allowing it to retract. However, this is not a problem in robots that can keep the swing leg angle using mechanical brakes, without costing much energy. Further, the walking patterns not preferred by humans (e.g., fast walking speed with small step-length, or slow walking with large step-length) might be demanded in robots for some scenarios or applications. Our study has indicated that the SLR does not benefit stability in these walking patterns. Thus, it's not necessary for a robot to implement SLR strategy in its controller when walking with these patterns. A non-retraction swing leg control strategy could be more stable and efficient in these walking patterns.

However, the results presented in this paper are preliminary. The conclusions are based on the simulation of 9 typical walking patterns in a simple model, rather than on solid theoretical analysis. Further systematic analysis and parametric studies are needed to identify the mechanism that leads to the different effects of SLR in human-preferred and non-preferred walking patterns.

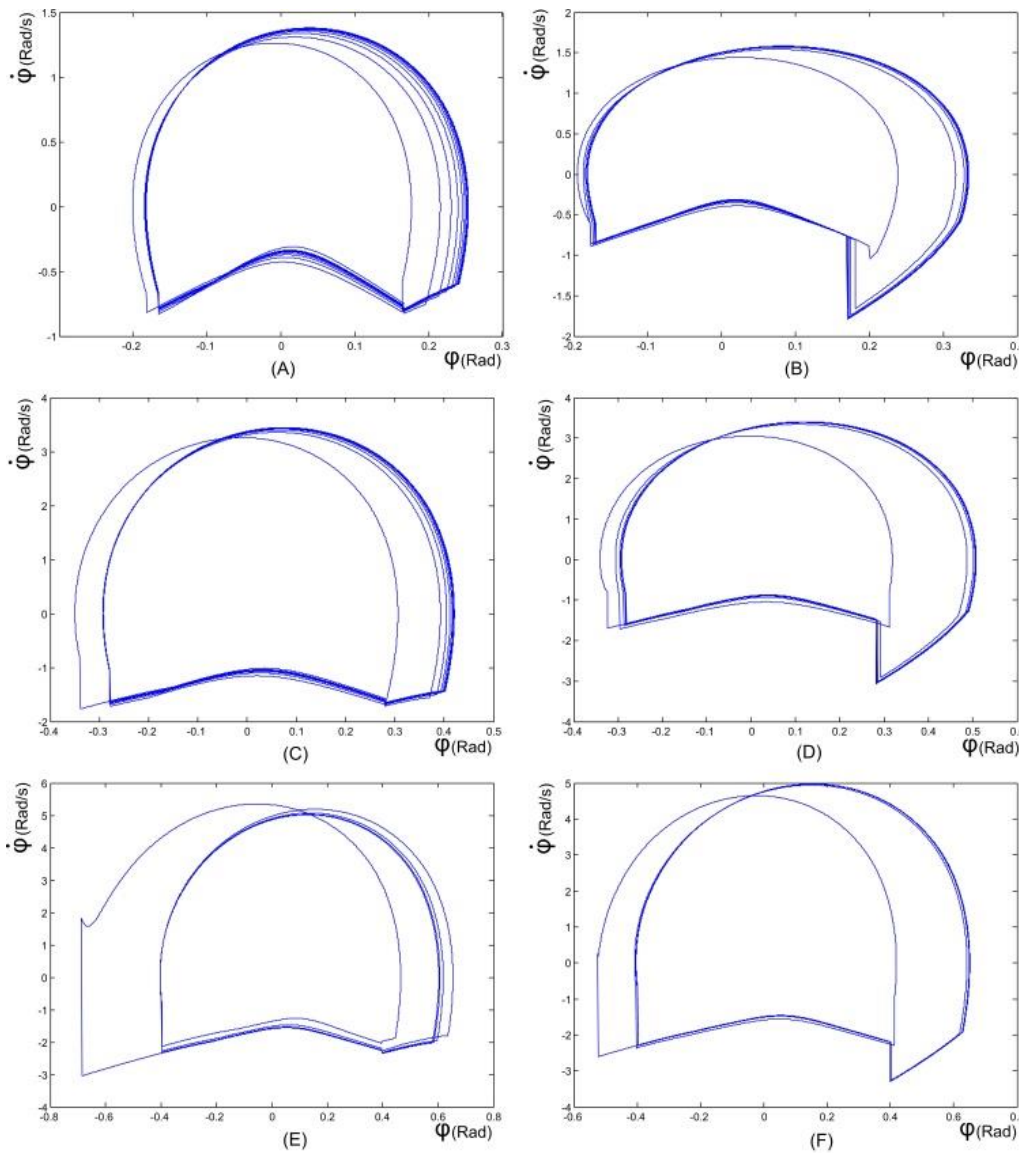


Fig. 4 The phase plots of one leg ( $\phi$ , see Fig. 1(A)) after walking down a step. Upper, middle, and lower rows are the human-preferred walking patterns of slow, medium, and fast speeds, respectively. Plots in the left column are without SLR, and plots in the right column are with mild SLR.

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